Moldova City Olympiad, Grade 12

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Problem 1 The polynomial $P(X) = aX^3 + bX^2 + cX + d$ takes integer values for $X = -1, 0, 1, 2$. Prove that $P(X)$ is an integer for any integer X.

Problem 2 Prove that

$$1 - e^{-\frac{\pi}{2}} < \int_0^{\frac{\pi}{2}} e^{-\sin x} \mathrm{d}x < \frac{\pi}{2}(1 - e^{-1}).$$

Problem 3 Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^{2007}e^x$. Prove that if $F(x)$ is a primitive for $f(x)$ then there exist 2008 unique integers $a_0, a_1, \ldots, a_{2007}$ so that

$$F(x) = [a_0 + a_1(x-1) + a_2(x-1)^2 + \ldots + a_{2007}(x-1)^{2007}]e^x + C.$$

Problema 4 Prove that the volume of any regular pyramid is less than $\frac{7}{17}$ of the cube of the lateral edge.