Moldova First Team Selection Test for IMO-BMO
2008, March 3

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Problem 1 Let p be a prime number. Solve in $\mathbb{N}_0 \times \mathbb{N}_0$ the equation $x^3 + y^3 - 3xy = p - 1$.

Problem 2 We say the set $\{1, 2, \ldots, 3k\}$ has property D if it can be split into disjoint triples, so that in each such triple, one number is the sum of the other two. Prove that

a) The set $\{1, 2, \ldots, 3324\}$ has property D.

b) The set $\{1, 2, \ldots, 3309\}$ hasn't property D.

Problem 3 Let $\Gamma(I, r)$ and $\Gamma(O, R)$ be the incircle and circumcircle, respectively, of triangle ABC. Consider all triangles $A_iB_iC_i$ which are simultaneously inscribed in $\Gamma(O, R)$ and circumscribed to $\Gamma(I, r)$. Prove that the centroids of the triangles $A_iB_iC_i$ lie on a circle.

Problem 4 A non-zero polynomial $S \in \mathbb{R}[X, Y]$ is called homogeneous of degree d if there is a positive integer d so that $S(\lambda x, \lambda y) = \lambda^d S(x, y)$ for any $\lambda \in \mathbb{R}$. Let $P, Q \in \mathbb{R}[X, Y]$ so that Q is homogeneous and P divides Q (that is $P|Q$). Prove that P is homogeneous too.