# Moldova Third Team Selection Test for IMO-BMO 2008, March 30

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**Problem 1** Determine a subset $A \subset \mathbb{N}^*$ having exactly 5 distinct elements, whose sum of squares equals their product.

**Problem 2** Let $p$ be a prime number and $k, n$ positive integers so that $\gcd(p, n) = 1$. Prove that $\binom{n \cdot p^k}{p^k}$ and $p$ are coprime.

**Problem 3** In triangle $ABC$ the bisector of $\angle ACB$ intersects $AB$ at $D$. Consider an arbitrary circle $O$ passing through $C$ and $D$ and not tangent to $BC$ or $CA$. Let $O \cap BC = \{M\}$ and $O \cap CA = \{N\}$.

a) Prove that there is a circle $S$ so that $DM$ and $DN$ are tangent to $S$ in $M$ and $N$ respectively.

b) Circle $S$ intersects $BC$ and $CA$ in $P$ and $Q$ respectively. Prove that the lengths of $MP$ and $NQ$ do not depend on the choice of circle $O$.

**Problem 4** A non-empty set $S$ of positive integers is said to be *good* if there is a coloring with 2008 colors of all positive integers so that no number in $S$ is the sum of two different positive integers (not necessarily in $S$) of the same color. Find the largest value $t$ can take so that the set $S = \{a + 1, a + 2, \dots, a + t\}$ is good, for any positive integer $a$.