Moldova Second Team Selection Test for IMO-BMO 2008, March 29

LaTeX'ed by Andrei Frimu

Problem 1 Find all solutions $(x, y) \in \mathbb{R} \times \mathbb{R}$ of the system:

$$\begin{cases} x^3 + 3xy^2 = 49, \\ x^2 + 8xy + y^2 = 8y + 17x. \end{cases}$$

Problem 2 Let $a_1, a_2, \ldots, a_n$ be positive reals so that $a_1 + a_2 + \ldots + a_n \leq \frac{n}{2}$. Find the minimal value of

$$\sqrt{a_1^2 + \frac{1}{a_2^2}} + \sqrt{a_2^2 + \frac{1}{a_3^2}} + \ldots + \sqrt{a_n^2 + \frac{1}{a_1^2}}$$

Problem 3 Let ω be the circumcircle of ABC and let D be a fixed point on (BC). X is a variable point on (BC), $X \neq D$. Denote by Y the second intersection of AX and ω. Prove that the circumcircle of triangle XYD passes through a fixed point.

Problem 4 Find the number of even permutations of $\{1, 2, \ldots, n\}$ which have no fixed points.